Option Valuation of Oil & Gas E&P Projects by Futures Term Structure Approach

March 9, 2007

Hidetaka (Hugh) Nakaoka
Former CIO & CCO of Itochu Oil Exploration Co., Ltd.
University of Tsukuba
Overview

1. Introduction
2. Valuation of Oil & Gas E&P Projects under Uncertainty of Recoverable Reserves
3. Term Structure of Futures Prices and Volatilities
4. Futures Term Structure Approach for Options Pricing
5. Empirical Study
6. Conclusion
1. Introduction
Objectives

(1) For a specific problem in E&P projects

Verifying the effect of the reserve risks, by introducing the uncertainty into the recoverable oil & gas reserves in the producing fields.

(2) For a common problem in real options

Introducing “Futures Term Structure Approach”, a new approach to track underlying asset value and its volatility for long term projects.
Backgrounds & Motivations

Reserve Risk

~ Uncertainty of the recoverable reserves have caused serious accounting problems to the super major oil companies.

Underlying Asset Prices & Volatility

~ Tracking problems in incomplete markets.
~ Maturity effect of the futures price volatility is supposed to affect the volatility of the underlying asset value of the long term project.
2. Valuation of Oil & Gas E&P Projects under Uncertainty of Developed Reserves
E&P Business Structure

- Exploration
- Development
- Production

Reserve Risk

Underlying Asset Price = Unit Value of Developed Reserves

Acreage Lease Contract

Options

Options

Asset Price Valuation

Cash Flow Valuation of Multi-steps Strategic Options

Best Fit to Real Options

~ A pioneer in this field.

~ Combination of option pricing theory and a market equilibrium model for the underlying asset value.

~ A comprehensive work on stochastic models for the exploration phase through the production phase.

**Reserve Risk:** considered only at the exploration phase

**Underlying Asset Value:** deemed to be 1/3 of spot price

**Volatility:** volatility of U.S. import oil price is applied
Reserve Risk in Oil & Gas E&P Projects

- Oil & gas accumulation is trapped in the reservoir rocks which normally lie several thousands meters deep underground.
- Recoverable reserves are always dependent upon geophysical conditions of the reservoirs such as formations, temperature, pressure, how to extract oil & gas and so on, which are the causes of the uncertainty.

Stochastic Model of Developed Reserves
2-(1) Underlying Asset Value under Uncertainty of Developed Reserves

Stochastic Process of Underlying Asset Value $U_t$

Assuming that $V_t$ (unit underlying asset value) follows the model of Paddock, Siegel and Smith (1988) and the developed reserves $B_t$ follows GBM process, the SDE of the underlying asset value of the Block $U_t = B_t V_t$ is derived from Ito’s lemma as follows;

$$\frac{dV_t}{V_t} = \alpha_v dt + \sigma_v dz_v$$  \hspace{1cm} $$\frac{dB_t}{B_t} = -\gamma dt + \sigma_B dz_B$$

$$\frac{dU_t}{U_t} = (\alpha_v - \gamma) dt + \sigma_U dz_U$$

$$\sigma^2_U = \sigma^2_v + \sigma^2_B$$

$$dz_U \cdot dz_B = \rho dt$$  \hspace{1cm} $$\rho = \frac{\sigma_B}{\sqrt{\sigma^2_v + \sigma^2_B}}$$

If $\rho_B = 0$, it will follow Paddock, Siegel and Smith (1988).
Probability Distribution Function of Reserve Volume $B_t$
2-(2) Development Option Value

Value of Undeveloped Reserve

* The partial differential equation for the value of the undeveloped reserve, \( Y_t \), in the development phase is yielded as follows by the arbitrage analysis subject to all the parameters assumed to be deterministic:

\[
\frac{\partial Y_t}{\partial t} + \frac{1}{2} \frac{\partial^2 Y_t}{\partial U_t^2} \sigma^2 U_t^2 + (r - \gamma) U_t \frac{\partial Y_t}{\partial U_t} - rY_t = 0
\]

* \( Y_t \) is described as an American call option under the following boundary conditions with the stopping time \( \tau \) and the strike price \( D^* \), which is the estimated development cost of the concession:

\[
Y(U_{\tau}^*, \tau) = (U_{\tau}^* - D^*)_+ = U_{\tau}^* - D^*
\]

\[
Y(0, t) = 0
\]

\[
Y(U_T, T) = (U_T - D^*)_+
\]
2-(3) Value of the Concession before Exploration

Value of Concession

\[ Y^*_t = \int Y(U_t(Q), t) dP(Q) \]

\[ dP(Q) = \begin{cases} p(0) & (Q = 0) \\ (1 - p(0)) f_E(Q) dQ & (Q \neq 0) \end{cases} \]

\[ Y(U_t(Q), t) : \text{the value of the undeveloped reserve.} \]
\[ Q : \text{the expected recoverable reserves estimated by the preliminary data.} \]
\[ P(Q) : \text{the probability distribution function of } Q. \]
\[ f_E(Q) : \text{the conditional probability density function of } Q, \text{ if } Q \text{ is not zero.} \]
Probability Density Function of $Q$ before Exploration
3. Term Structure of Futures Prices and Volatilities
Maturity Effect of Volatility

**Samuelson hypothesis (1965)**

“The volatility of futures price returns should increase as time to maturity decreases.”

(“maturity effect of volatility”)

Maturity effect is observed and modeled in such commodity futures prices as crude oil, agricultural commodities, and metals. Little evidence is observed in the financial asset prices.

**Term Structure Model in Our Empirical Study**

Schwartz (1997) Two-Factor Model
Term Structure of Volatility of WTI Daily Rate of Return

Volatility (Daily)

Delivery Month
Estimated Volatility Term Structure of WTI Futures Price Rate of Return

Volatility

Delivery Month
4. Futures Term Structure Approach for Options Pricing
Futures Term Structure Approach

Underlying Asset Price of the Project

The unit underlying asset price of the project $V$ is described as unit expectation of an integral of the discounted present value of the net cash flow $f_t(S_t, B_t)$ under the risk neutral probability measure subject to $S_t$ being the crude oil spot price at time $t$ and $F_t(u)$ being the futures price with the maturity $u$;

$$V = \frac{1}{B} \tilde{E} \left[ \int_0^T e^{-rt} f_t(S_t, B_t) dt \middle| S_0 = S, B_0 = B \right]$$

Net Cash Flow $f$

Assuming that $B_t$ is deterministic and $f_t(S_t)$ is affine as described as $f_t(S_t) = a_t S_t + b_t$, then

$$\tilde{E} [ f_t(S_t) | S_0 = S ] = a_t \tilde{E} [ S_t | S_0 = S ] + b_t = a_t F_0(t) + b_t = f_t(F_0(t))$$
Underlying Asset price $V$

$$V = \frac{1}{B} \int_0^T e^{-rt} f_t(F_0(t)) dt$$

“Futures Term Structure Approach”

If the futures prices can be inferred up to the maturity of the project life, then the underlying asset price can be estimated as the discounted value of the deterministic net cash flow.

1. The maturity effect of the volatility can be incorporated.
2. Fiscal terms, OPEX, etc. for each project can be incorporated.
5. Empirical Study
5-(1). Underlying Asset Price of “Project X”
Empirical Case Study ～”Project X”～

Basic Terms of “Project X”

- Proven recoverable oil & gas reserves: 1 billion barrels
- Option to develop (postpone): 5 years
- Construction cost: $2.59 billion
- Risk-free interest rate: 6.0% per annum
- Project life: 27 years
- Market price of the products: linked with WTI futures price
Tracking of Project X Underlying Asset Price by “Futures Term Structure Approach”

Long Horizon WTI Futures Price

Long horizon WTI futures prices are inferred from the data of WTI futures prices from January 1990 through March 2001, applying the Kalman filter to the Schwartz (1997) two-factor model.

Underlying Asset Price $V$

Applying the above prices to the financial model of Project X, time series of underlying asset prices are estimated. As a result, the following parameters are estimated, assuming geometric Brownian motion:

- Expected rate of return: 2.6% per annum
- Volatility: 10.8% per annum
- Average price: $4.74 per barrel
Volatility of Project X

Traditional Risk Measurement (WTI Front Month Model)

As a common industrial practice, E&P project owners normally evaluate their project risk by the volatility of the rate of return of either spot oil prices or futures front month prices. Volatility of WTI futures front month prices during January 1990 and March 2001 is 28.1% per annum.

“Futures Term Structure Approach“

Volatility of Project X during the same period is 10.8% per annum.

Comparison of the Real Options Valuation

We compare the two approaches in options valuation at different levels of underlying asset prices, applying a binomial model.
Valuation of Project X as an American Call Option

<table>
<thead>
<tr>
<th>Underlying Asset Price</th>
<th>2.00</th>
<th>2.50</th>
<th>3.00</th>
<th>3.50</th>
<th>4.00</th>
<th>4.50</th>
<th>5.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term Structure Model</td>
<td>0.21</td>
<td>0.61</td>
<td>1.09</td>
<td>1.58</td>
<td>2.08</td>
<td>2.58</td>
<td>3.08</td>
</tr>
<tr>
<td>WTI Front Month Model</td>
<td>0.50</td>
<td>0.90</td>
<td>1.29</td>
<td>1.71</td>
<td>2.18</td>
<td>2.66</td>
<td>3.14</td>
</tr>
</tbody>
</table>
5-(2) Development Option Value under Uncertainty of Developed Reserves
Volatility of Underlying Asset Price $U$
under Uncertainty of Developed Reserves

$\sigma_U^2 = \sigma_V^2 + \sigma_B^2$

$\sigma_V$: Volatility of Unit Value of Developed Reserve
$\sigma_B$: Volatility of Developed Reserve Volume

Table 1. Volatility Change in Underlying Asset Price with Change in Volatility of Reserve Volume

<table>
<thead>
<tr>
<th>$\sigma_B$</th>
<th>0.0%</th>
<th>2.5%</th>
<th>5.0%</th>
<th>7.5%</th>
<th>10.0%</th>
<th>12.5%</th>
<th>15.0%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_V$</td>
<td>10.8%</td>
<td>10.8%</td>
<td>10.8%</td>
<td>10.8%</td>
<td>10.8%</td>
<td>10.8%</td>
<td>10.8%</td>
</tr>
<tr>
<td>$\sigma_U$</td>
<td>10.8%</td>
<td>11.1%</td>
<td>11.9%</td>
<td>13.2%</td>
<td>14.7%</td>
<td>16.5%</td>
<td>18.5%</td>
</tr>
<tr>
<td>□</td>
<td>0.00</td>
<td>0.23</td>
<td>0.42</td>
<td>0.57</td>
<td>0.68</td>
<td>0.76</td>
<td>0.81</td>
</tr>
</tbody>
</table>
## Impact on Development Option Value by Uncertainty of Developed Reserves

### Table 2. Impact on Development Option Value by Uncertainty of Developed Reserves

<table>
<thead>
<tr>
<th>Underlying Asset Price</th>
<th>2.00</th>
<th>2.50</th>
<th>3.00</th>
<th>3.50</th>
<th>4.00</th>
<th>4.50</th>
<th>5.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_B = 0%$</td>
<td>210</td>
<td>610</td>
<td>1,090</td>
<td>1,580</td>
<td>2,080</td>
<td>2,580</td>
<td>3,080</td>
</tr>
<tr>
<td>$\Delta_B = 5%$</td>
<td>230</td>
<td>620</td>
<td>1,090</td>
<td>1,580</td>
<td>2,080</td>
<td>2,580</td>
<td>3,080</td>
</tr>
<tr>
<td>$\Delta_B = 10%$</td>
<td>290</td>
<td>660</td>
<td>1,100</td>
<td>1,590</td>
<td>2,080</td>
<td>2,580</td>
<td>3,080</td>
</tr>
<tr>
<td>$\Delta_B = 15%$</td>
<td>350</td>
<td>730</td>
<td>1,150</td>
<td>1,620</td>
<td>2,100</td>
<td>2,590</td>
<td>3,080</td>
</tr>
</tbody>
</table>

(million dollars)
Impact on Development Option Value by Uncertainty of Reserves

- Development Option Premium ($MM)
- Underlying Asset Price ($/bbl)
- Variation in Reserve Uncertainty: σB = 0%, 5%, 10%, 15%
5-(3) Value of Concession under Uncertainty of Developed Reserves
## Value of Concession under Uncertainty of Developed Reserves

Table 3. Value of Concession under Uncertainty of Developed Reserves

<table>
<thead>
<tr>
<th>Underlying Asset Price ($/bbl)</th>
<th>2.00</th>
<th>2.50</th>
<th>3.00</th>
<th>3.50</th>
<th>4.00</th>
<th>4.50</th>
<th>5.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta U = 10.8%$ (MM$)$</td>
<td>104</td>
<td>266</td>
<td>455</td>
<td>650</td>
<td>849</td>
<td>1,049</td>
<td>1,249</td>
</tr>
<tr>
<td>$\Delta U = 18.5%$ (MM$)$</td>
<td>155</td>
<td>306</td>
<td>478</td>
<td>665</td>
<td>858</td>
<td>1,054</td>
<td>1,251</td>
</tr>
</tbody>
</table>

(million dollars)
Value of Concession under Uncertainty of Developed Reserves

- **No Reserve Risk** (Volatility 10.8%)
- **With Reserve Risk** (Volatility 18.5%)
6. Conclusion
Conclusion

(1) In this paper, we derive stochastic equations for the underlying asset value of the E&P project under the uncertainty of the developed reserves. We verify that such reserve risks can give significant impact on the development options value and the value of the concession written over the underlying asset.

(2) In order to solve tracking problems of the underlying asset prices, we introduce a new practical approach of “Futures Term Structure Approach”, which verifies that the volatility of the oil & gas E&P project should be much smaller than those estimated by the traditional risk measurement due to the maturity effect of the volatility of the futures prices.