Implied Loss Distribution, Term Structure of Correlation Skew and Dynamic Modeling of Credit Portfolio

Quantitative Analytics
Global Credit Derivatives Group
Barclays Capital

David Li
+1 212 412 3551
david.li@barcap.com
Outline

- Current Portfolio Credit Derivative Market
- Copula function approach to credit portfolio modeling
- Extension of Gaussian Copula Functions: Mixture of Copula Function; Gaussian extension
- Implied Loss distribution
- CDO and CDO^2 Pricing using Implied loss distribution
- Dynamic Model of Portfolio Loss distribution
Some of the latest Credit Portfolio Products

- CDO^2 with cross subordination
- CDO of long and short credit or CDO^2 with long and short tranches; CDO of global credits
- Forward CDOs
- CDO with changing subordination levels or amortizing underlying credits
- Tranchelets and non-standard index tranches
- ABX index and tranche: HEL
### Market Quotes: CDX, March 01, 2006

<table>
<thead>
<tr>
<th>CDX.5 (12/10) Ref (42) delta</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-3% 33 1/4 - 33 5/8</td>
<td>22.0x</td>
</tr>
<tr>
<td>3-7% 91 - 93</td>
<td>5.5x</td>
</tr>
<tr>
<td>7-10% 19 - 21</td>
<td>1.5x</td>
</tr>
<tr>
<td>10-15% 9 - 11.5</td>
<td>0.7x</td>
</tr>
<tr>
<td>15-30% 3 - 5.5</td>
<td>0.3x</td>
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<table>
<thead>
<tr>
<th>CDX.5 (12/12) Ref (51) delta</th>
<th>Change</th>
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<tbody>
<tr>
<td>0-3% 51 1/8 - 52 5/8</td>
<td>14.5x</td>
</tr>
<tr>
<td>3-7% 221 - 226</td>
<td>9.5x</td>
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<tr>
<td>7-10% 36 - 38</td>
<td>2.2x</td>
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<tr>
<td>10-15% 17 - 20</td>
<td>1.1x</td>
</tr>
<tr>
<td>15-30% 5.5 - 6.5</td>
<td>0.4x</td>
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</table>

<table>
<thead>
<tr>
<th>CDX.5 (12/15) Ref (64) delta</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-3% 58 3/4 - 59 1/4</td>
<td>7.5x</td>
</tr>
<tr>
<td>3-7% 587 - 595</td>
<td>12.0x</td>
</tr>
<tr>
<td>7-10% 97 - 101</td>
<td>4.7x</td>
</tr>
<tr>
<td>10-15% 48 - 51</td>
<td>2.2x</td>
</tr>
<tr>
<td>15-30% 12 - 15</td>
<td>0.6x</td>
</tr>
</tbody>
</table>
Default Correlation: The Joy of Copulas

- We first know the marginal distribution of survival time for each credit.
- We need to construct a joint distribution with given marginals and a correlation structure.
- Copula function used in multivariate statistics can be used.
- The correlation parameters used in copula function can be interpreted as the asset correlation between two credits used in CreditMetrics.
What is a Copula Function?

- Function that join or couple multivariate distribution functions to their one-dimensional marginal distribution functions
- For m uniform r. v., U1, U2, ...., Um
  \[ C(u_1,u_2,\ldots,u_m) = \Pr[U_1 \leq u_1, U_2 \leq u_2, \ldots, U_m \leq u_m] \]
- Suppose we have m marginal distributions with distribution function \( F_i(x_i) \)
- Then the following defines a multivariate distribution function
  \[ F(x_1, x_2, \ldots, x_m) = C(F_1(x_1), F_2(x_2), \ldots, F_m(x_m)) \]
How do we simulate the default time in the normal copula function framework?

- Simulate a joint normal distribution $Y_i$ with a given correlation matrix $\Sigma$
- Translate $Y$ into a uniform random variable $Z$
- Use each credit curve to get survival time for each credit

$$T_i = F_i^{-1}(\Phi(Y_i))$$
Efficient Credit Portfolio Simulation

- Importance Sampling
  - For single name we should shift the normal mean from 0 to 0.865
- Quasi Monte Carlo
  - It does not work very well for high dimension; but it would help tremendously if we use it in conjunction with the reduction of dimension
- Reduction of Dimensionalities
  - One correlation – one factor model
  - Inter and intra industry correlation – (n + 1) factor model where n is the number of industry groups
- Fast Fourier Transformation Approach (FFT)
- Recursive Algorithm (conditional convolution) and other Approximation (conditional approximation)
Excess Loss Distribution

(125 names, 55 bps, 40% recovery rate)
Tranche Loss as an Option on the Total Portfolio Loss

Tranche Loss v.s. Total Loss

- Super Senior
- Senior
- Mezzanine
- Mezzanine Subord
- Equity
Base Correlation

- For each CDO tranche loss leg could be deemed as a call spread on the loss distribution, long a call with the strike equal to detachment amount and short a call with a strike equal to the attachment amount.

- Implied correlation should be quoted against only equity tranches with different detachment points. This would give a consistent framework.

- The hedge ratio would be different in two cases: using base correlation and not using base correlation.
Base Correlation and Index Level
Various Extension of Basic Gaussian Copula Model

Problem: How to price bespoke portfolio?
Various mapping approaches, and various Gaussian extensions

- Student, Marshal-Olkin, Negative Inverse Gaussian
- Gaussian Extension: Andersen and Sidenus
- Composite Basket Model
- Gaussian Mixture
Mixture of Copula Functions

- Basic Idea: Correlation is small in good times and large in bad times
- One solution is to make correlation random
- Using copula function we know that the mixture of copula function is still a copula function

\[ C(U) = \int C(u \mid \rho) d\nu (\rho) \]
Mixture Copula Function: Discrete Case

\[
\sum_{j=1}^{m} \alpha_j \cdot C_j \left( u_1, u_2, \ldots, u_m ; \rho_j \right) \\
\sum_{j=1}^{m} \alpha_j = 1 ,
\]
Gaussian Mixture

- We have three Gaussian copula functions and each with one constant correlation parameter rho
- We also have two independent mixing parameters alpha1 and alpha 2. \( \alpha_3 = 1 - \alpha_1 - \alpha_2 \)
- We can use this approach to calibrate to the index market with 5 frequently traded tranches
- The calibration is relatively stable. We obtain three correlation parameters around 0%, 25% and 90% and the mixing parameters around 60%, 20% and 20%.
Implied Loss Distribution

An equity tranche with tranche size $K$ can be valued as follows:

$$
E \left( L_T (K) \right) = \int_0^K S_{L_P} (x) \, dx
$$

$$
\frac{\partial E \left( L_T (K) \right)}{\partial K} = S_{L_P} (K) = \Pr \left[ L_P > K \right]
$$

$$
\frac{\partial^2 E \left( L_T (K) \right)}{\partial K^2} = -f_{L_P} (K)
$$

Using market index tranche spreads and base correlation we can obtain the implied loss distribution of CDS index portfolio.
Loss Distribution Comparisons

CDX Loss Distribution: Implied, GM, and 15% Flat Gaussian

- Market
- Gaussian Mixture
- Flat 15%

Prob (L > K) vs. strike
Comparison of Leverage Ratios

Comparision of CDX Leverage Ratio

Leverage Ratio

Tranches

Gaussian Mixture
Gaussian
Pricing CDO^2-Type Transactions

- Calculate implied loss distribution
- Obtain model implied loss distribution
- Create a mapping between the market implied loss distribution and model implied loss distribution
- Using this mapping to price all CDOs and CDO^2
- Create a balance between matching to market and also using an economic plausible model
Term Structure of Base Correlation

<table>
<thead>
<tr>
<th>Strike</th>
<th>20-Dec-08</th>
<th>20-Dec-10</th>
<th>20-Dec-12</th>
<th>20-Dec-15</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0%</td>
<td>11.82%</td>
<td>8.94%</td>
<td>6.00%</td>
<td>2.72%</td>
</tr>
<tr>
<td>6.0%</td>
<td>18.95%</td>
<td>18.16%</td>
<td>18.75%</td>
<td>5.81%</td>
</tr>
<tr>
<td>9.0%</td>
<td>22.93%</td>
<td>23.81%</td>
<td>28.73%</td>
<td>13.68%</td>
</tr>
<tr>
<td>12.0%</td>
<td>24.72%</td>
<td>28.71%</td>
<td>36.47%</td>
<td>19.19%</td>
</tr>
<tr>
<td>22.0%</td>
<td>35.96%</td>
<td>40.19%</td>
<td>57.34%</td>
<td>34.43%</td>
</tr>
</tbody>
</table>
Loss-Grid Approach

Strike Direction

Time Direction
Dynamic Models Based on Portfolio Loss

- To model total portfolio loss only
- Using either short rate type model for instantaneous loss ratio or forward rate model for forward loss ratio
- Functional form, senior tranche, calibration issues
- Sensitivities, going from index to bespoke

- Ultimate Model: replication Model?
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