Two events, A and B are *independent* if the probability of A and B, that is, the probability of their intersection, $P(A \cap B) = P(A) P(B)$. This is automatic but uninteresting if $P(A) = 0$ or 1, likewise for B. If $0 < P(A), P(B) < 1$, then an equivalent statement is that $P(A|B) = P(A)$, that is, the probability of A *given* B is the probability of A, alternatively $P(B|A) = P(B)$. The point here is that the occurrence or lack of it of A, alternatively B, does not influence the occurrence or lack of it of the other. Thus, A, B independent if, and only if (iff) $A, B^c; A^c, B; A^c, B^c$ are also independent (pairs of events), where $A^c$ means “not A,” the complement of A. A common mistake is to confuse two events being independent and their being disjoint. A and B are disjoint if $A \cap B = 0$, that is, the occurrence of one precludes that of the other.
Pictorially, that is, with Venn diagrams,

**Independent Events**

![Venn diagram for independent events](image)

**Disjoint Events**

![Venn diagram for disjoint events](image)

One common example of independent events is that of, say, “heads” and “tails” on two successive tosses of the same coin. Disjoint events would be the events “heads on the first toss” and “tails on the first toss.”
Any set of events is said to be *independent* if the probability of the intersection of any (finite) number of them is the product of their respective probabilities. Same story as before when any event is replaced by its complement. The definition of independence here requires that for *any* finite subcollection of them (or their complements), the probability of the intersection is the product of the probabilities.

Here’s a thought exercise that you can do on your own.

Imagine that you have a fair coin and that you toss it 9 times. The first, second, … ninth events are the respective results of the tosses. That is,
$A_i$ is the event [“heads” on the $i$th toss] for $i = 1, 2, \ldots 9$. The event $A_{10}$ is “heads” if the previous 9 tosses had an even total number of heads; otherwise, $A_{10}$ is “tails.” Then any 9 of the 10 events taken together are independent. However, the entire group of them is not. If you know the outcomes of any nine events you can tell with certainty the results of the remaining one! The lesson here is that even our most elementary of concepts is a bit tricky. We will see very soon that inferences for such as *prospective case-control studies* and other *observational* studies depend very much on what appears on the right hand side of conditional probability statements, that is, upon what the assumptions are that underlie such inferences. On the other hand, such subtleties make our subject matter both fun and challenging.